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Application of anomaly matching to SU(N) spin chains and generalization of Haldane conjecture

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Based on 1805.11423 with Tin Sulejmanpasic.

Haldane ('83) shows the low-energy EFT of SU(2) spin chain,

$$\hat{H} = J \sum_i \hat{S}_i \cdot \hat{S}_{i+1} \quad (J>0),$$

is given by $\mathbb{CP}^1 \left(= \frac{SU(2)}{U(1)}\right)$ non-linear sigma model

$$\mathcal{L} = \int |(d + ia)\phi|^2 + i\frac{\theta}{2\pi}\int da$$

$\phi \in \mathbb{C}^2$, $|\phi|^2 = 1$ $\quad a : U(1)$ gauge field $(= i\bar{\phi} d\phi)$

with $\hookrightarrow U = [\phi, \varepsilon \cdot \bar{\phi}] \in SU(2)$.

$\theta = 2\pi S$.

He then conjectures ← Lieb-Schultz-Mattis thm (Affleck, Lieb '86)

- $S = \frac{1}{2}, \frac{3}{2}, \cdots$: Gapless ($\to$ SU(2), WZW model)
- $S = 1, 2, \cdots$: Gapped.

Motivation

Generalize the Haldane conjecture to SU(N) anti-ferrom. spin chain.

Setup

[Bykov ('12) and] Lajko, Wamer, Mila, Affleck ('17) showed that the low-energy EFT of SU(N) spin chain,

$$\hat{H} = J_1 \sum_i \hat{S}_i \cdot \hat{S}_{i+1} + J_2 \sum_i \hat{S}_i \cdot \hat{S}_{i+2} \quad (J_1, J_2 > 0),$$

is given by $\frac{SU(N)}{U(1)^{N-1}}$ non-linear sigma model

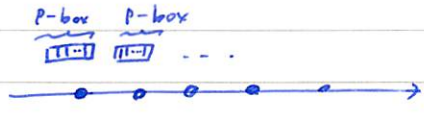
$$\mathcal{L} = \sum_{\ell=1}^{N} \Big\{ \int |(d + ia_\ell)\phi_\ell|^2 + i\frac{\theta_\ell}{2\pi}\int da_\ell + \lambda \int (\bar{\phi}_{\ell+1} \cdot d\phi_\ell) \wedge (\bar{\phi}_\ell \cdot d\phi_{\ell+1}) \Big\}.$$

Constraint

- $\phi_\ell \in \mathbb{C}^N$, $\bar{\phi}_\ell \cdot \phi_m = \delta_{\ell m}$ (i.e. $U = [\phi_1, \cdots, \phi_N] \in U(N)$) ← $\phi_N = \varepsilon\, \bar{\phi}_1 \cdots \bar{\phi}_{N-1}$.
- $a_N = -a_1 - \cdots - a_{N-1}$ and $\det[U] = 1$. ($\Rightarrow$ We can set $\theta_N = 0$).

Lattice symmetry requires

$$\theta_\ell = \frac{2\pi\ell}{N} p.$$



We'll study the phase structure of SU(3) spin chain.

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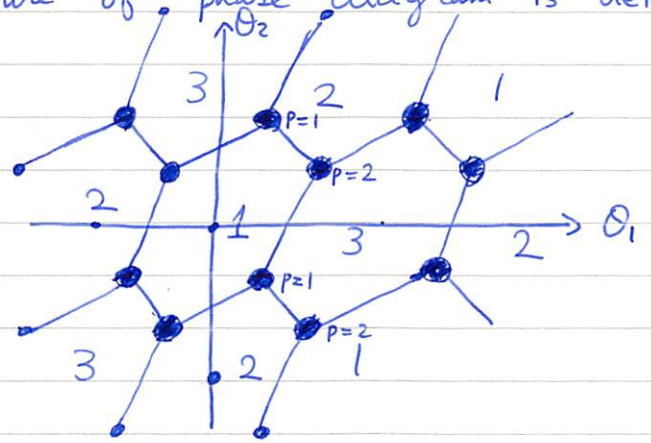
Result (1805.11423, YT, Sulejmanpasic)

- Global structure of phase diagram is determined by symmetry:



1, 2, 3: Different SPT orders protected by PSU(3).

●: PSU(3) × $\mathbb{Z}_3$ with 't Hooft anomaly

$\Rightarrow$ { · Trimerized phase · Gapless ($\to$ SU(3)$_1$ WZW).

In this talk, we restrict our attention to PSU(3) × $\mathbb{Z}_3$ symmetric point with 't Hooft anomaly, and discuss the consequence of anomaly matching.

Symmetry

- PSU(3) spin symmetry.

This maps the spinon fields $\phi_\ell \in \mathbb{C}^3$ as

$$\phi_\ell \longmapsto V \cdot \phi_\ell \qquad (V \in SU(3)).$$

Because of U(1) gauge inv., the symm. with faithful rep. is $PSU(3) = \frac{SU(3)}{\mathbb{Z}_3}$.

- $\mathbb{Z}_3$ lattice translation symmetry.

$$\phi_\ell \longmapsto \phi_{\ell+1}$$

Then.

$$\Delta \mathcal{L} = \sum_\ell i \frac{(\theta_{\ell+1} - \theta_\ell)}{2\pi} \int d a_\ell .$$

Since $\sum_\ell a_\ell = 0$ (by constraint), if $\theta_{\ell+1} - \theta_\ell = \text{const} \pmod{2\pi}$,

$$\Delta \mathcal{L} = \sum_\ell i \frac{(\text{const})}{2\pi} \int da_\ell = 0 \mod 2\pi$$

$\Rightarrow$ For $\theta_\ell = \frac{2\pi \ell}{N} p$ $(p \in \mathbb{Z})$, $\mathbb{Z}_N$ is symmetry.

(cf. $\theta_{\ell+1} - \theta_\ell = \frac{2\pi p}{N}$ $(\ell = 1, \cdots, N-1)$

$\theta_1 - \theta_N = \frac{2\pi p}{N} - 2\pi p$.)

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· $PSU(3) \times \mathbb{Z}_3$ anomaly

Introduce the background gauge field for $PSU(3) = \frac{SU(3)}{\mathbb{Z}_3}$:

- A : $SU(3)$ 1-form gauge field
- B : $\mathbb{Z}_3$ 2-form gauge field.

$$\mathcal{L}_{gauged} = \sum_\ell \Big\{ \int |(d + i a_\ell + iA)\,\phi_\ell|^2 + i\frac{\theta_\ell}{2\pi}\int (da_\ell + B) + \lambda \int \{(\phi_{\ell+1}\cdot(d+iA)\phi_\ell) \wedge (\phi_\ell \cdot (d+iA)\phi_{\ell+1})\} \Big\}.$$

Perform the $\mathbb{Z}_3$ lattice translation, then

$$\Delta \mathcal{L}_{gauged} = i\sum_\ell \frac{(\theta_{\ell+1}-\theta_\ell)}{2\pi}\int (da_\ell + B)$$
$$= i\sum_\ell \frac{1}{2\pi}\cdot\frac{2\pi p}{N}\underbrace{\int (da_\ell + B)}_{=\partial} - i\frac{1}{2\pi}\cdot 2\pi p \int (da_N + B)$$
$$= -ip\int B. \qquad \text{mod } 2\pi i$$

$$\Rightarrow \mathbb{Z}_N: Z_p[(A,B)] \longrightarrow Z_p[(A,B)]\, e^{-ip\int B}.$$

In other words,

$$Z_p[\underbrace{(A,B}_{PSU(N)\text{ gauge field}}, \underbrace{A_\chi}_{\mathbb{Z}_N\text{ gauge field}})]\, e^{-i\frac{Np}{2\pi}\int_{3D} A_\chi \wedge B} \quad \text{is gauge inv.}$$

Anomaly matching claims

- $\mathbb{Z}_3$ is spontaneously broken (= trimerized phase), or
- System is gapless.

We can further say that $SU(3)_1$ WZW model has the same anomaly:

$$\mathcal{L}_{WZW,k} = \int_{M_2} \mathrm{tr}[\,|(g^\dagger dg)|^2] + i\frac{2\pi k}{24\pi^2}\int_{M_3}\mathrm{tr}[(g^\dagger dg)^3]. \qquad (g\in SU(3))$$

Symmetry: $\frac{SU(3)_L \times SU(3)_R}{\mathbb{Z}_3} \supset \frac{SU(3)_V}{\mathbb{Z}_3} \times (\mathbb{Z}_3)_{L(=R)}$.

$g \mapsto \underbrace{V\cdot g\cdot V^\dagger}$, $\quad g \mapsto \underbrace{e^{\frac{2\pi i}{N}} g}$.

Define gauged WZ 3-form as

$$J = \frac{1}{24\pi^2}\mathrm{tr}[(g^\dagger D g)^3] + \frac{1}{8\pi^2}\mathrm{tr}[(gDg^\dagger)(dA+A^2+dA_\chi) - (g^\dagger Dg)(dA+A^2)].$$

Then $dJ = \frac{N}{(2\pi)^2} d(A_\chi \wedge B)$

$\rightsquigarrow$ $\mathcal{L}_{gauged\,WZW,k} - \underbrace{\frac{Nk}{(2\pi)^2} A_\chi \wedge B}$ is gauge inv.

same anomaly - for $k = \pm 1$. $\therefore$ $SU(3)_1$ WZW matches anomaly.

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